Observation of Partially Suppressed Shot Noise in Hopping Conduction

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We have observed shot noise in the hopping conduction of two dimensional carriers confined in a p-type SiGe quantum well at a temperature of 4K. Moreover, shot noise is suppressed relative to its "classical" value 2eI by an amount that depends on the length of the sample and carrier density, which was controlled by a gate voltage. We have found a suppression factor to the classical value of about one half for a 2 μm long sample, and of one fifth for a 5 μm sample. In each case, the factor decreased slightly as the density increased toward the insulator-metal transition. We explain these results in terms of the characteristic length ($\simeq 1\mu$m in our case) of the inherent inhomogeneity of hopping transport.

Shot noise, which is a manifestation of the particle nature of the electric current, has lately received much attention [1] because it can yield information complementary to that obtained from conductance measurements. It is most pronounced when the current is formed by statistically independent charges tunneling through a single potential barrier of low transparency, in which case the noise power spectral density, S, is equal to the Schottky, or classical, value of $2qI$, where q is the value of the charge and I the average current. This proportionality has been employed, for instance, to determine the effective charge in superconducting transport [2] and in the fractional quantum Hall effect [3]. In more general cases, when the motion of the charges is not independent from each other, the value of S has an additional factor F, the so called Fano factor. Except when negative differential conductance occurs [4], the Fano factor is in the range $0 < F < 1$, meaning that shot noise is then partially suppressed. Knowledge of the degree of suppression sheds light at the microscopic level on the conduction mechanisms of a specific system.

By now the noise characteristics of ballistic, diffusive, and chaotic transport have been established, as well as those of resonant tunneling and single-electron tunneling. For example, in diffusive conductors, whose size along the current direction is smaller than the inelastic scattering length, it is $F = 1/3$. In the opposite limit, that is, when the scattering length is much shorter than the length of the sample, F approaches zero, and in macroscopic metallic conductors shot noise is completely suppressed.

Although the 1/f noise properties of hopping conduction have also been elucidated [5], surprisingly, little is known about shot noise for such a well studied transport mechanism [6], which has regained interest in connection with the metal-insulator transition recently observed in Si MOSFETs [7] and other two-dimensional (2D) systems, including SiGe quantum wells [8]. In this Letter we report the observation in a 2D hopping conductor of shot noise that is only partially suppressed, and we introduce a model that in spite of its simplicity can explain our experimental results.

If in hopping conduction, where electrons tunnel assisted by phonons between localized states created by the random impurity potential, the determinant factor were the inelastic scattering length, then, given the smallness of this length, shot noise should be zero. On the other hand, since the process involves tunneling through potential barriers, which insures the discrete nature of the current, one could naively assume that shot noise should have the full $2qI$ value. A closer look reveals a more complex situation.

In a simple one-dimensional system in which electrons tunnel through N identical barriers, the Fano factor is $F = 1/N$. When, like in hopping, tunneling occurs between single electron states, depending on their occupancy, shot noise suppression can be a different function of N [9]. Since the equivalent resistances of the various hops are exponentially different from each other and only the most resistive hop ("bottle neck") determines the current, it could be argued that effectively $N = 1$. However, in real quasi one-dimensional hopping [10], in which there is a maximum resistance obtainable (hard hop), the effective N should be the number of hard hops along the sample length, as in the case of identical barriers.

In a 2D system, hopping conduction can be seen as occurring through a network of one-dimensional chains connected to each other at certain nodes, as is normally done in percolation theory, where the network is modeled by resistors of exponentially different values, out of which the most conductive subnetwork (critical percolation subnetwork) is selected [6]. The characteristic size of this subnetwork is the length beyond which the sample is homogeneous, and its nodes are such that each chain contains only one resistor with the largest resistance (hard hop). Even this simpler subnetwork is still complicated enough as to make it difficult to guess, let alone to calculate, what the effective F will be.

To answer experimentally this question we chose a 2D hole system confined in a modulation-doped SiGe well.

The heterostructure, grown by molecular beam epitaxy on a n-Si substrate, consists (from the substrate up) of 4300Å of Si boron-doped at 1×10^{18} cm^{-3}, 225Å of undoped Si, 500Å of $Si_{0.8}Ge_{0.2}$ (quantum well), 275Å of undoped Si, and finally 725Å of Si boron-doped at 1×10^{18} cm^{-3}. The 2D hole density and the in-plane resistivity were controlled by a voltage Vg applied to an Al Schottky gate deposited on the top layer. The heterostructure was processed into samples with gate width of 50μm and length (along the current direction) of either 2μm or 5μm. The noise measurements were done with the samples immersed in liquid He. At $T = 4K$ even for the smallest Vg used the samples had resistance per square much larger than the quantum resistance $h/2e^2$, so that they were always in the insulating regime.

Current through the sample was produced by applying a *dc* bias and a small *ac* signal, Vin, to a $1M\Omega$ load resistor. The voltage drop across the sample, Vsd, was measured simultaneously with the *ac* signal and the noise, using a lock-in technique and a spectrum analyzer. The $1/f$-noise contribution was reduced by doing the measurements at high frequencies up to 100kHz, which demanded minimizing lead capacitance and required placing a preamplifier inside the cryostat and very close to the sample. The preamplifier, with an output resistance of about 100Ω, was a commercial low power switching MOSFET, connected in source follower configuration, as in [11]. To avoid heating, the *dc* current through the preamplifier was kept at 1mA, enough to get an amplification coefficient of 0.9. The input impedance (defined by the resistance of the sample in parallel with the load resistor and parasitic capacitances) of the preamplifier was proportional to the preamplifier's transfer function, TF $(Vout/Vin)$, whose real and imaginary components were determined with the *ac* signal applied to the load.

Figure 1(a) shows the measured output voltage noise for a 2μm long sample as a function of gate voltage, in the absence of any in-plane current (zero bias) and for three different frequencies (20 kHz, 50 kHz, and 80 kHz). The origin of this voltage is thermal noise. As expected, the voltage noise spectral density follows the real part of the impedance and of the transfer function, shown in Fig. 1(b). The background preamplifier noise, seen at small Vg (small sample resistance) in Fig.1a, has a 1/f dependence. Taking into account this noise, about $6nV/\sqrt{Hz}$ for $f_0 = 80kHz$, we confirmed that the measured noise is indeed thermal noise at $T = 4K$ (insert in Fig.1b). When the sample resistance is much larger than the load resistance (at high gate voltage) the TF saturates. From the saturation value we determined a parallel to the sample capacitance of about $2pF$, which is mainly the gate-drain capacitance of the preamplifier.

The transfer function exhibits small but noticeable oscillations at $Vg \simeq 0.5V$, as illustrated in Fig. 1(b). Their presence suggests that the sample is close to the mesoscopic size, where conductance is not self-averaged but depends on a particular spatial configuration of the fluctuation potential. In the language of percolation theory, we can say that those oscillations reveal that the length of the sample and the size of the critical subnetwork are comparable [12] [13].

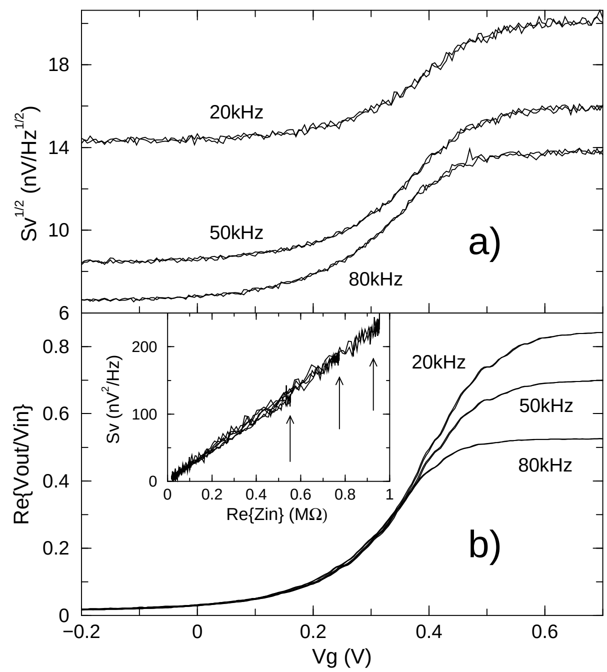

FIG. 1. The output voltage noise for a 2μm long sample (a) and the real part of the transfer function (b) as a function of gate voltage, at three frequencies. The insert depicts the power spectral density noise, obtained from (a), as a function of the input impedance determined from (b). Three curves are superimposed on the graph, one for each of the three frequencies at which the voltage noise was measured. For each frequency, the maximum resistance at which the voltage noise was measured is indicated by an arrow, starting with the highest frequency on the left.

For a fixed gate voltage, the current noise was obtained by measuring the voltage noise spectral density as a function of the current. The transfer function, measured simultaneously, was then used to calculate the current noise spectral density at the preamplifier input. The dependence of the current noise on current at $Vg = 0.5V$ is shown in Fig.2, for $f = 20$, 50 and $80kHz$. The insignificance of the thermal noise (the residual noise density at $Vsd = 0$) is a consequence of the fact that the sample's resistance is larger than the load resistance. Indeed, the theoretical thermal noise is $4k_BT/(1M\Omega) \simeq 0.2 \times 10^{-27}A^2/Hz$, a number that is consistent with the $I = 0$ limit of Fig. 2.

The signatures of shot noise – linear dependence on current and independence of frequency – are evident in the figure. In view of the strongly non-linear dependence of the current on Vsd (insert in Fig. 2), it could seem surprising that the proportionality between noise and current is maintained in a large current range. This proportionality suggests (as validated below) that even at

the highest voltage the in-plane electric field is still weak enough as not to modify the critical percolation network, and indicates that the measured shot noise is not sensitive to possible field-induced variations of the hopping percolation paths. From that proportionality, a value $F = 0.59$ is obtained for the Fano factor.

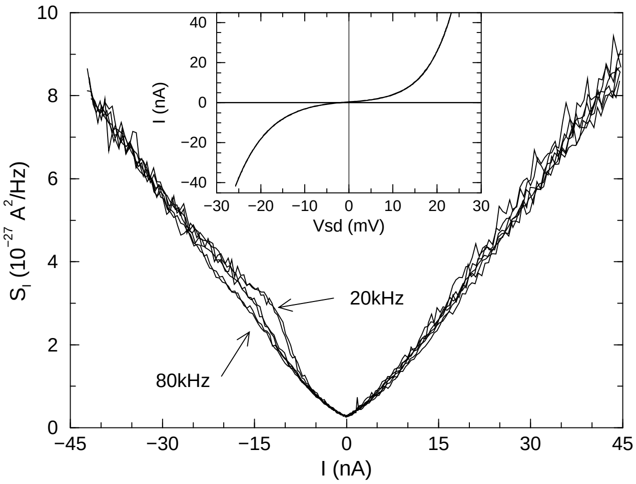

FIG. 2. Current noise spectral density as a function of the current in a 2μm long sample subjected to a 0.5V gate voltage, showing the proportionality between current and noise for the entire current range. From the slope of the curves a Fano factor F = 0.59 is deduced. The three superimposed curved are for measurements at the same three frequencies of Fig.1. The arrows point to the hump that appears for a current of about -10 nA, which is most pronounced at the lowest frequency. Th insert shows the current-voltage characteristic, measured simultaneously with the noise.

It is noticed in Fig.2 that at $I = -10nA$ there is a hump in the noise spectral density, which is larger at lower frequency. This is a signature of random telegraph noise in mesoscopic structures [14], also seen before in hopping transport [15].

The Fano factor does not depend very strongly on gate voltage, as shown in Fig.3 (top curves). When Vg decreases from $Vg = 0.55V$ to $0.2V$ F drops from 0.61 to 0.43, which is still roughly one half of the classical value in this range of Vg. Since the smaller the gate voltage the larger the in-plane conductance, for $Vg = 0.2V$ thermal noise dominates at low current, thus the curvature in the noise characteristic observable in Fig. 3. The transition from thermal to shot noise occurs above $2kT/e$.

Figure 3 also illustrates the dependence of shot noise on the length of the current path. Similar measurements to those on the $2\mu m$ sample are shown for a $5\mu m$ sample. In this case, the variation of shot noise with Vg has the same trend as before, but the change is smaller. Most significant, however, is that shot noise is much more suppressed in the longer sample, in which the measured Fano factor is F = 0.2, that is, shot noise is 1/5 of its classical value.

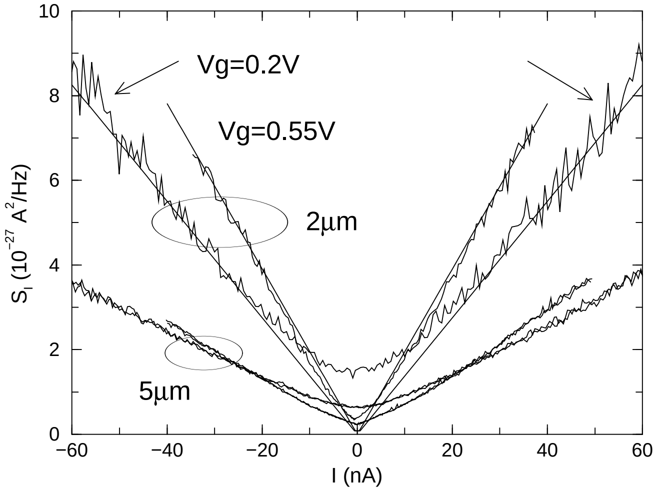

FIG. 3. Dependence of shot noise on gate voltage for samples whose length was either $2\mu m$ or $5\mu m$. For each set of curves (marked by an ellipse) the gate voltages were 0.2V and 0.5V. The straight lines superimposed on the curves of the $2\mu m$ set give Fano factors of 0.61 and 0.43 for $Vg = 0.55V$ and $0.2V$, respectively.

We can explain our results if we assume that there is a characteristic scaling length $L_0 \simeq 1\mu m$ in 2D hopping for both samples, such that the Fano factor is just the ratio of that length to the length of the sample, $F = L_0/L$. It is reasonable to assume also that this scale is a characteristic of the homogeneity of the sample, which is the distance between hard hops of the critical percolation subnetwork. Then, the trend for noise suppression on Vg reflects the fact that hopping becomes more uniform as the sample is driven towards the insulator-metal transition.

To justify these assumptions we can take into account the fact that when an electric field is applied, only hard hops along the field direction are modified (e.g., decrease their resistances) and thus the network is separated into a set of equivalent parallel chains. In this case, the total shot noise would be that of a single chain, which, as we have seen, will have a Fano factor inversely proportional to the number of hard hops in the chain.

The distance between hard hops in the percolation subnetwork can be obtained using [18]:

$$L_0 = l(T)\Big(\frac{T_0}{T}\Big)^{\frac{\nu}{d+1}}, \tag{1}$$

where $l(T)$ is the characteristic hopping length in the zero-field limit, T_0 is a characteristic temperature inversely proportional to the density of states at the Fermi level and to the localization radius a, d is the effective dimensionality of the system (d=1 for hopping with Coulomb gap and d=2 for 2D hopping), and ν is the critical index of the correlation radius, which is about 1.3 for a 2D system. In turn, the hopping length can be estimated within the percolation model, in which the non-linear conductance G(E,T) is written [16–18] as

$$G(E,T) = \frac{I}{Vsd} = G(0,T)exp\Big(\frac{eEl(T)}{k_BT}\Big), \qquad (2)$$

where E is the electric field. (The other symbols have their usual meaning.) This expression is only valid in the low-field regime, that is, when $eEa < k_BT$. The hopping length depends on temperature as

$$l(T) = a\Big(\frac{T_0}{T}\Big)^{\frac{1}{1+d}}. \qquad (3)$$

When Eq. (2) was used in combination with the experimental $I - Vsd$ characteristics (Fig. 2, insert) a value of $l \simeq 0.08\mu m$ was obtained for the hopping length of the $2\mu m$ sample at T = 4K. The localization radius was estimated to be 100-130Å from Eq. (3) and the experimental temperature dependence of the zero-field conductance in the range $2K < T < 30K$. The 30Å variance of a reflects the difficulty in discerning experimentally between 1/2 and 1/3 for the exponent in Eq. (3). This estimation of the localization radius is consistent with a fluctuation potential created by interface impurities separated about 200Å from each other (surface density of $2 \times 10^{11} cm^{-2}$ [19]) and validates our low-field assumption since for $Vsd = 30mV$ (insert, Fig. 2) it is $Ea \simeq 0.15meV < k_BT/e \simeq 0.36meV$.

Finally, from Eq.(1) we get either $L_0 \simeq 0.8\mu m$ or $\simeq 1.2\mu m$, depending, again, on whether we use $d = 1$ or 2. A similar analysis for the data on the $5\mu m$ sample yielded a value of $1\mu m$ for the inter-node distance. This result tells us that if two nodes were separated by $1\mu m$, than there would be two hard hops in a $2\mu m$ sample and five such hops in a $5\mu m$ sample. Consequently, according to the above discussion, the corresponding Fano factor in the shot noise formula should be 0.5 and 0.2, respectively. These values are in excellent agreement with the experimental results.

Although these results have been obtained for a 2D hole gas, they should be general to any other system in the hopping regime. It also follows from our results that by decreasing the sample length even further one could obtain full shot noise, corresponding to tunneling through only one hard hop along the current direction. Interestingly, a further decrease in length and in temperature should cause a transition to resonant tunneling transport, such as resonant tunneling through impurities [20]. The only calculation available in such a regime for the shot noise is for tunneling through one impurity, done recently [21], in which $F = 3/4$. We hope that the experiments presented here will stimulate calculations in the entire hopping conduction regime.

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